Global representation rings

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October 2015

Loyola Universtiy Chicago

AMS Sectional Meeting

Celebrating the 75th birthday of Donald S. Passman.

Where

This talk is available online at computo.fismat.umich.mx/∼valero

in long and short version.

See also J. of Algebra, Volume 441, 1 November 2015, Pages 426–440.

Character table

Definition

Throughout this lecture, let G denote a finite group. The character table of G is a square matrix, whose columns are indexed by the conjugacy classes of elements g of G, and whose rows are indexed by isomorphism classes of simple $\mathbb{C}G$-modules S, and the entry at such column and row is given by $X_S(g)$, which is the trace of the matrix by which g acts on S.

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Table of marks

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The table of marks of G is a square matrix, whose columns and rows are indexed by the conjugacy classes of subgroups of G, and whose entry H, K (where H and K are subgroups of G) is denoted $\varphi_H(G/K)$, called the mark of H in G/K, and equals the number of fixed points of G/K under the action of H.

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The representation ring

Definition

Take the Grothendieck group on the characters of the simple $\mathbb{C}G$-modules (up to isomorphism). Note that the sum of characters can also be given by the direct sum of representations. Define a product using the tensor product of characters. This gives a ring structure, called the representation ring of the group G, or the Green ring of G.

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The Burnside ring

Definition

The Burnside ring of the group G is the subring of $\mathbb{Z}^n$ (where n is the number of conjugacy classes of subgroups of G) spanned by the columns of the table of marks. Note that if two rings have the same tables of marks, then their Burnside rings are isomorphic: the converse is an open problem. The Burnside ring can alternatively be defined using the Grothendieck group on the transitive G-sets, and defining a multiplication using the cartesian product of G-sets. Any ring homomorphism f from $B(G)$ to $\mathbb{Z}$ is a mark, that is, there exists a fixed subgroup H of G such that $f(G/K) = (G/K)^H = \varphi_H(G/K)$ for all K.

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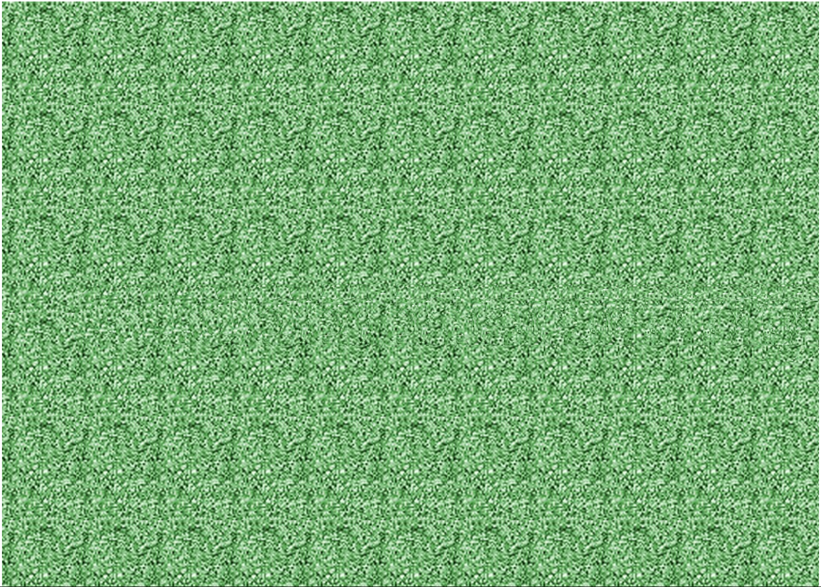
Combining characters and marks

Remark

Now let's look at the big picture

Combining characters and marks



Graded module

Definition

Let X be a finite G-set. We say that a $\mathbb{C}G$-module V is X-graded if

$$V = \bigoplus_{x \in X} V_x, V_x \neq 0$$

where we require that $gV_x = V_{gx}$ for all $x \in X$.

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Bundle category

Definition

Let $\mathcal{L}$ be the category whose objects are pairs (X, V) where X is a finite G-set, and V is an X-graded $\mathbb{C}G$-module. The morphisms in $\mathcal{L}$ from (X, V) to (Y, W) are pairs of morphisms (α, f) where $\alpha : X \longrightarrow Y$ is a morphism of G-sets, $f : V \longrightarrow W$ is a morphism of $\mathbb{C}G$-modules and $f(V_x) \subseteq W_{\alpha(x)}$ for all x in X.

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Examples

- define the sum of (X, V) and (Y, W) to be $(X \sqcup Y, V \oplus W)$
- define the product of (X, V) and (Y, W) to be $(X \times Y, V \otimes W)$
- the zero element is $(\emptyset, 0)$
- the unity is $(\cdot, \mathbb{C})$, where $\mathbb{C}$ denotes the trivial $\mathbb{C}$-module

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$[H, W]$

Definition

Let H be a subgroup of G and W a $\mathbb{C}H$-module. We denote the isomorphism class of the object $(G/H, i_H^G(W))$ by $[H, W]$.

$T(G)$

Definition

From the category $\mathcal{L}$ we construct its Grothendieck group and create a ring, denoted $T(G)$.

Mackey formula

Theorem

We have a product formula:

$$[H,W][K,U] = \sum_{x\in[H\backslash G/K]} [H\cap {}^xK, r^H_{H\cap {}^xK}(W) r^{{}^xK}_{H\cap {}^xK}({}^xU)]$$

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Global representation ring $Д(G)$

Definition

We define the global representation ring of the group G as the quotient

$$Д(G) = T(G)/\left\langle [H, V_1 \oplus V_2] - [H, V_1] - [H, V_2] \right\rangle .$$

We also write $[H, V]$ to denote elements of $Д(G)$. This ring is similar to previous constructions, such as Witherspoon's ring of G-vector bundles, Nakaoka's ring, and Hartmann and Yalçin's generalized Burnside ring, but with some important differences.

Global representation ring Д(G) 10-48

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Basis of $Д(G)$

Definition

The elements $[H, S]$ with S simple form a basis of $Д(G)$ over the integers.

Ring homomorphisms from $T(G)$ to $\mathbb{C}$

Definition

Let H be a subgroup of G and b in H. We define a ring homomorphism $\mathcal{S}_{H,b} : T(G) \longrightarrow \mathbb{C}$ by

$$\mathcal{S}_{H,b}([K,S]) = \sum_{\substack{x \in [G/K] \\ H \leq^{x} K}} X_{{}^{x}S}(b)$$

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Each $\mathcal{S}_{H,b} : T(G) \longrightarrow \mathbb{C}$ induces a ring homomorphism $\mathcal{S}_{H,b} : Д(G) \longrightarrow \mathbb{C}$. In fact, every ring homomorphism from $Д(G)$ to $\mathbb{C}$ is one of these.

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Serge Bouc and Robert Boltje

This construction is strongly linked to Bouc's biset functors and Boltje's plus construction.

One ring automorphism on $Д(G)$

Definition

Define a ring automorphism σ on $Д(G)$ by sending $[K, V]$ to $[K, V^*]$. For every $\mathcal{S}_{H,b}$ we have that $\mathcal{S}_{H,b} \circ \sigma = \mathcal{S}_{H,b^{-1}}$.

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Theorem

$$\mathcal{S}_{H,1}([K,S]) = \varphi_H(G/K)\dim S$$

$$\mathcal{S}_{H,b}([K,\mathbb{C}]) = \varphi_H(G/K)$$

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Table of species

Definition

The square table whose rows are indexed by $[H, b]$ and columns by $[K, S]$ and whose entries are $\mathcal{S}_{H,b}([K, S])$ is the table of species. This matrix consists of blocks (indexed by conjugacy classes of subgroups of G) along the diagonal, and zeros below them. The last block on the diagonal is the character table of G.

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An application

Theorem

The order of G is an even integer if and only if there exists a ring homomorphism $f : Д(G) \longrightarrow \mathbb{C}$ whose image lies inside the real numbers and such that f is not a mark (that is, $f \neq S_{H,1}$ for all H).

An application

Theorem

Let G be an arbitrary finite group. If there exists a non-trivial primitive idempotent in $\amalg(G)$ then there exists a ring homomorphism from $\amalg(G)$ to the real numbers which is not a mark.

This theorem is in fact equivalent to the famous Feit-Thompson theorem: any group of odd order must be soluble.

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The prime ideals of $\text{Д}(G)$

Let n be the order of the group G, and w a primitive complex n-th root of unity. For each $S_{H,b}$, let $P_{H,b}$ denote its kernel, and $Im_{H,b}$ its image inside $\mathbb{Z}[w]$. Let m denote a maximal ideal of $\mathbb{Z}[w]$, let $\overline{S^m_{H,b}}$ be the composition of $S_{H,b}$ followed by the quotient map to $\mathbb{Z}[w]/m$, and let $P_{H,b,m}$ be the kernel of $\overline{S^m_{H,b}}$. Let p denote the characteristic of $\mathbb{Z}[w]/m$.

Theorem

The spectrum of $\text{Д}(G)$ consists of all the ideals of the form $P_{H,b}$ and $P_{H,b,m}$.

The prime ideals of $\Delta(G)$

Let n be the order of the group G, and w a primitive complex n-th root of unity. For each $\mathcal{S}_{H,b}$, let $P_{H,b}$ denote its kernel, and $Im_{H,b}$ its image inside $\mathbb{Z}[w]$. Let m denote a maximal ideal of $\mathbb{Z}[w]$, let $\overline{\mathcal{S}^m_{H,b}}$ be the composition of $\mathcal{S}_{H,b}$ followed by the quotient map to $\mathbb{Z}[w]/m$, and let $P_{H,b,m}$ be the kernel of $\overline{\mathcal{S}^m_{H,b}}$. Let p denote the characteristic of $\mathbb{Z}[w]/m$.

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The prime ideals of $\mathbb{D}(G)$

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The spectrum of $\mathbb{D}(G)$ consists of all the ideals of the form $P_{H,b}$ and $P_{H,b,m}$.

The connected components

Theorem

Let H be a perfect subgroup of G, that is, $H = O^s(H)$. Let X_H consist of all ideals $P_{K,b}$ and $P_{K,b,m}$ where $O^s(K)$ is conjugate to H in G. Then X_H is a connected component of the spectrum of $Д(G)$, and any connected component is one of the X_H with H perfect.

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The primitive idempotents

Theorem

For every $K \leq G$ and $c \in K$, we have that

$$e_{K,c} = \frac{1}{|N_G(K) \cap C_G(c)|} \sum_{\substack{(H,S) \\ c \in H \leq K \\ S \in Irr(H)}} \mathcal{X}_S(c)\mu(H,K)[H,S]$$

Here we take all subgroups H of G, and a set of representatives S of the irreducible $\mathbb{C}H$-modules.

We also have that the minimum positive integer t such that $te_{K,c}$ belongs to $\mathbb{A}_w(G) := \mathbb{Z}[w] \otimes \mathbb{A}(G)$ is $|N_G(K) \cap C_G(c)|$.

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Isomorphisms of species

These are isomorphisms that send the basis $[H, S]$ in $Д(G)$ to the basis in $Д(Q)$. The existence of such an isomorphism implies that the groups G and Q share many invariants (for example, they have isomorphic centres).

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Final words

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Thank you!