Bol. Soc. Mat. Mexicana (3) Vol. 10, 2004

STABLE PARTITIONS AND ALPERIN'S WEIGHT CONJECTURE FOR THE SYMMETRIC GROUPS IN CHARACTERISTIC TWO

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Abstract. Alperin's weight conjecture for the symmetric groups has been proved using an enumeration of the weights and the simple modules (see [2]), but so far there is no explicit way to associate weights with simple modules. Based on data obtained using an algorithm for finding weights for small symmetric groups in characteristic two (see [16]), we put forward a combinatorial conjecture which, if true, would provide explicit bijections between weights and irreducible modules for the symmetric groups in characteristic two. We prove some results towards the proof of this combinatorial conjecture.

1. Introduction

Alperin's weight conjecture is one of the most important and difficult open problems in the representation theory of finite groups. This conjecture has already been established for several families of groups, but some of these proofs are just enumerations of the sets of weights and irreducibles, with no explicit correspondence between them. Such is the case of the symmetric groups. Alperin and Fong proved in [2] that Alperin's conjecture holds for the symmetric groups, so we know that the number of weights for kS_n equals the number of simple kS_n-modules, where k is a field of characteristic $p > 0$. In [16] the second author used Brauer quotients to assign weights to the simple modules parameterized by three infinite families of 2-regular partitions. Using computer software written in GAP (see [9]), he used Brauer quotients to give an explicit bijection bewteen weights and irreducible modules for kS_n for $n \leq 9$. This information was gathered in one Table of Partitions, whose rows are indexed by the weight subgroups of all the symmetric groups, and whose columns are indexed by the triangular partitions. This table is in Section 4. In this section we also observe some remarkable properties of this Table of Partitions, and put forward a conjecture. This conjecture is a stronger version of a reformulation of Alperin's conjecture for the symmetric groups in characteristic two.

In Section 2 we define weights and state Alperin's conjecture. In Section 3 we give James' construction of the irreducible kS_n-modules in characteristic p. At the end of this section we also define skew-hooks and cores of partitions, which are needed to determine the blocks of the irreducibles. Section 4 has the Table

2000 *Mathematics Subject Classification*: 20 (Group theory).

Keywords and phrases: group representation, Alperin's conjecture, weight, Brauer quotient, symmetric group, partition.

of Partitions and our proposed conjecture. Section 5 has the results we have towards a proof of this conjecture.

2. Alperin's Conjecture

We give the definition of weight and formulate Alperin's Conjecture in its most general form. We mention some classes of groups for which it is known to be valid (including the symmetric groups) and we note the possible advantages of a combinatorial proof, that is, an explicit bijection between weights and irreducible modules.

Throughout this section, G will be a finite group, p a prime number, and k a splitting field for G in characteristic p. All our modules will be finite dimensional over k.

Definition (2.1). A *weight* for G is a pair (Q, S) where Q is a p-subgroup and S is a simple module for $k[N_G(Q)]$ which is projective when regarded as a module for $k[N_G(Q)/Q]$.

Remark (2.2). Since S is $k[N(Q)]$-simple and Q is a p-subgroup of $N_G(Q)$, it follows that Q acts trivially on S, so S is also a $k[N_G(Q)/Q]$-module and the definition makes sense. Moreover, S is $k[N_G(Q)/Q]$-simple as well.

Remark (2.3). If we replace S by an isomorphic $k[N_G(Q)]$-module we consider this the same weight, and we make the same identification when we replace Q by a conjugate subgroup (so that the normalizers will be conjugate, too).

Now we can formulate the main problem that we shall discuss in this section.

Conjecture (2.4) (Alperin's conjecture). The number of weights for G equals the number of simple kG-modules.

A stronger version of the preceding statement is that there is a bijection within each block of the group algebra.

Definition (2.5). If (Q, S) is a weight for G, then S belongs to a block b of $N_G(Q)$ and this block corresponds with a block B of G via the Brauer correspondence; hence we can say that the weight (Q, S) *belongs* to the block B of G so the weights are partitioned into blocks.

Conjecture (2.6) (Alperin's Conjecture, Block Form). The number of weights in a block of G equals the number of simple modules in the block.

This version of the conjecture implies the original one, as it can be obtained by summing the equalities from the stronger conjecture over the blocks. This stronger conjecture has been proved when G is a:

Finite group of Lie type and characteristic p (Cabanes, [8]).
Soluble group (Okuyama, [14]).
Symmetric group (Alperin and Fong, [2]).
$GL(n, q)$, p odd and p does not divide q (Alperin and Fong, [2]).
$GL(n, q)$, $p = 2$ and q odd (An, [3]).

The conjecture has also been checked in a variety of other cases (see [4], [5], [6], [7], etc.).

Alperin and Fong's proof in the case of symmetric groups was just an observation of a numerical equality which did not suggest a deeper reason for the relationship. For finite groups in general one does not expect to have any canonical bijection between weights and simple modules; as a matter of fact, Alperin himself says this is unlikely (see [1], p 369). For groups of Lie type in their defining characteristic there is a canonical bijection (described in [1]). Since symmetric groups and groups of Lie type have such strong connections in their representation theory, it is reasonable to ask whether there is some canonical bijection in the case of symmetric groups.

If true, Alperin's conjecture would imply a number of known results, until now unrelated (see [1]). It is also reasonable to expect that if an explicit bijection can be given to prove it, this may reveal new connections between simple kG-modules and weights; there are many results known about the former, and the latter are related to the blocks of defect zero, which are not as easy to deal with as the simple modules. In fact, this is really the true importance of Alperin's conjecture in that it provides a connection between the blocks of defect zero and the set of all simple modules. More specifically, Alperin's conjecture has been shown by Knörr and Robinson [13] to be equivalent to a statement which expresses the number of blocks of defect zero of a group in terms of the number of p-modular irreducibles of sections of the group of the form $N_G(P)/P$, $P \leq G$ a p-subgroup. These latter numbers are easy to compute, since by a theorem of Brauer the number of p-modular irreducibles of a group equals the number of p-regular conjugacy classes.

3. Some important kS_n-modules

We define the modules M^λ, S^λ and D^λ following James [11]. The simple kS_n-modules, as is well known, can be parameterized by certain partitions of n called p-regular, where p is the characteristic of the field k. Moreover, it is possible to construct each simple module from its associated partition. We end this section with the definition of p-core. In this section n is a natural number, k is a field of characteristic $p > 0$ and λ is a partition of n.

Definition (3.1). A λ-*tableau* is one of the $n!$ arrays of integers obtained by replacing each node in the partition λ by one of the integers $1, 2, \ldots, n$, allowing no repeats. If t is a tableau, its *row stabilizer*, R_t, is the subgroup of S_n consisting of the elements which fix all rows of t setwise. The *column stabilizer* of t, denoted C_t, is the subgroup of S_n consisting of the elements which fix all columns of t setwise. The *signed column sum* of t, denoted κ_t, is the element of kS_n given by

$$\kappa_t := \sum_{\pi \in C_t} (-1)^{\text{sign}(\pi)} \pi.$$

We define an equivalence relation on the set of λ-tableaux by $t_1 \sim t_2$ if and only if $\pi t_1 = t_2$ for some $\pi \in R_{t_1}$. The *tabloid*, $\{t\}$ containing t is the equivalence class of t under this relation. The kS_n-module $M^\lambda = M_k^\lambda$ is the vector space over k whose basis elements are the various λ-tabloids. The *polytabloid*, e_t, associated with the tableau t is given by

$$e_t := \kappa_t \{t\}.$$

The *Specht module*, $S^\lambda = S_k^\lambda$ for the partition λ is the submodule of M^λ spanned by polytabloids (this is indeed a kS_n-module).

We also define an S_n-invariant, symmetric, non-singular *bilinear form* $\langle\ ,\ \rangle$ on M^λ, whose values on pairs of tabloids is given by

$$\langle t_1, t_2\rangle := \begin{cases} 1 & \text{if } t_1 = t_2, \\ 0 & \text{if } t_1 \neq t_2. \end{cases}$$

The partition λ is *p-singular* if it has at least p rows of the same size; otherwise, λ is *p-regular*. The module $D^\lambda = D_k^\lambda$ is defined as

$$D^\lambda := S^\lambda/(S^\lambda \cap S^{\lambda\perp})$$

where λ is a p-regular partition.

Theorem (3.2) (James). *As λ varies over p-regular partitions of n, D^λ varies over a complete set of inequivalent irreducible kS_n-modules. Each D^λ is self-dual and absolutely irreducible. Every field is a splitting field for S_n.*

For a proof of this result, see [11].

Definition (3.3). Let λ be a partition. A *skew-hook* is a connected part of the rim of λ which can be removed to leave a proper diagram. The *r-core* of λ is the partition obtained by removing all possible skew-hooks of size r from λ (this is a well-defined partition, that is, the order in which we remove the skew-hooks does not matter). A *brick* is a skew-hook of size 2 (compare to the definition of *domino* in [10]). Recall that if $\lambda = (\lambda_1, \ldots, \lambda_t)$ is a 2-regular partition, then its rows must be of different sizes, i.e. $\lambda_1 > \lambda_2 > \cdots > \lambda_t$.

Theorem (3.4) (Nakayama's Conjecture). *Let α and β be p-regular partitions of n, and let k be a field of characteristic $p > 0$. Then D^α and D^β lie in the same block of kS_n if and only if α and β have the same p-cores.*

For a proof of this result see [12], Thm 6.1.21.

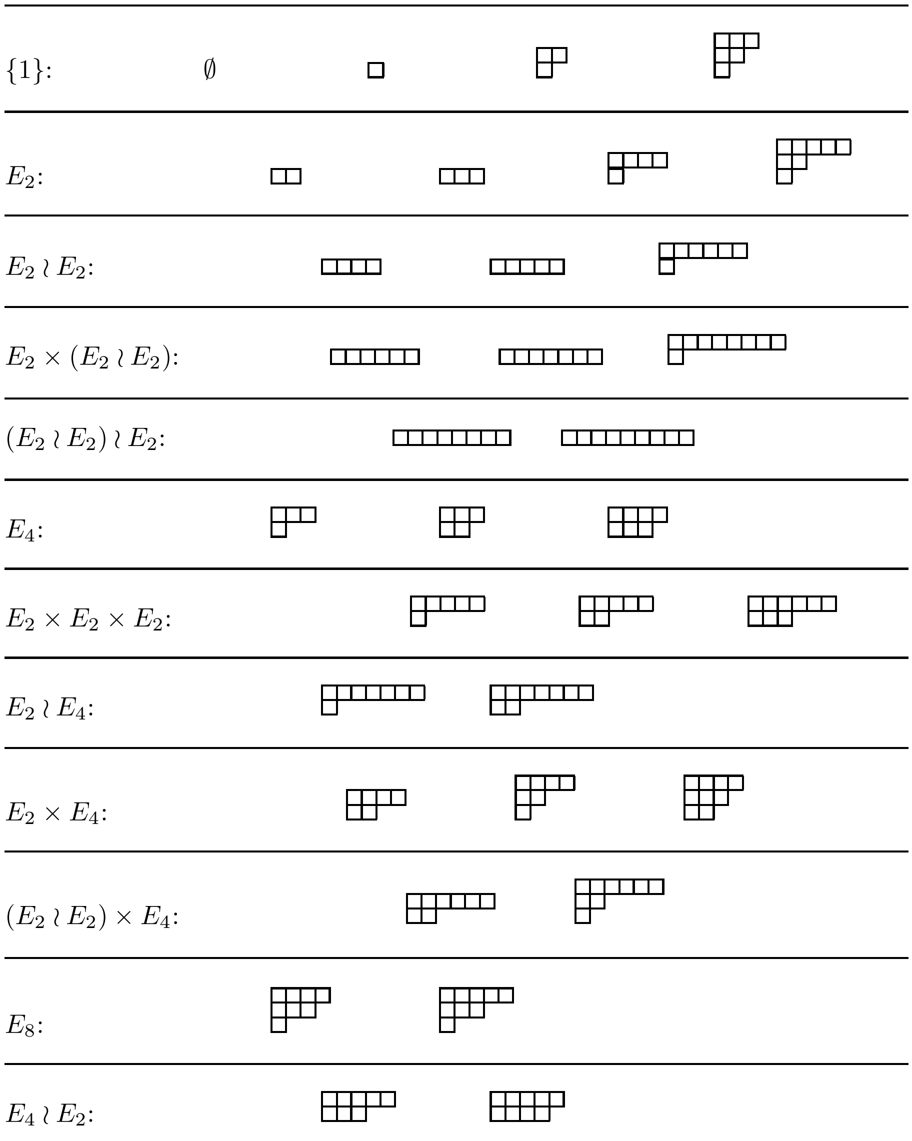
4. Table of partitions

Since the number of weights for the symmetric group S_n equals the number of simple kS_n-modules, one can define explicit bijections between weights and irreducibles. Given any such possible bijection, it is natural to ask whether there is a pattern hidden in its construction. The following table of partitions shows the pattern in the case of the partial correspondence described in [16].

Notice that the fact that the characteristic is 2 implies that each weight (Q, S) for S_n is uniquely determined by its weight subgroup Q. Indeed, the quotient $N_{S_n}(Q)/Q$ is in general the semidirect product of some copies of $GL(m_i, 2)$ and S_m (see [2]). Since any simple projective module for the direct product of $GL(m_i, 2)$ is S_m-invariant and since S_m has at most one simple projective module, Clifford theory tells that (Q, S) is determined by Q.

Each weight subgroup of S_n is used to index one row of the table. In order to determine the weight associated to a simple S_n-module V, one should first look up in the table the partition that parameterizes V, and locate the subgroup Q of S_n that indexes its row. By the previous remark, this subgroup can be completed to a unique weight (Q, S) for S_n. This is the weight that corresponds to the irreducible module V.

The correspondence for these small values of n was determined using Brauer quotients. For these specific irreducible modules and weight subgroups, we constructed their Brauer quotients (see [16] for their definition) and determined which ones were simple and projective. This algorithm worked for all but one of the simple modules of kS_n for $n \leq 9$ and k a field of characteristic two. We used software written in GAP (see [9]) to make these computations. The routines we used were written by Peter Webb and Luis Valero-Elizondo.



Two-regular partitions

Notice the following facts about this table of partitions:

1. The trivial subgroup indexes a row that consists of all triangular partitions (we included the triangular partitions of size 0 and 1 for completeness).
2. Each weight subgroup Q appears for the first time inside a symmetric group S_n where n is such that Q has no fixed points on the set $\{1, \ldots, n\}$. Moreover, if a 2-regular partition of size m appears in a row indexed by the group Q, then Q is a weight subgroup of S_m.
3. The first partition of every row has empty 2-core. The second partition has 2-core of size 1, the third has 2-core of size 3 and the fourth has 2-core of size 6. In other words, the 2-core of every partition along the i-th column is the i-th triangular partition (where $\emptyset$ is the first triangular partition).
4. For every row, all partitions λ in that row are such that the difference of the size of λ minus the size of its 2-core is constant.
5. Along every row, each partition is contained in the one to its right.

Item 1 is just stating the well-known fact that in characteristic 2, the only symmetric groups with simple projective modules are the S_t with t a triangular number, and that such modules are parameterized by the corresponding triangular partitions. Item 2 is proved implicitly in [2].

It is rather straightforward to come up with the following conjecture:

Conjecture (4.1). It is possible to arrange all 2-regular partitions in an infinite table satisfying the five conditions mentioned above.

Note that the existence of an infinite table of partitions satisfying conditions 1, 2, 3 and 4 is equivalent to the block version of Alperin's conjecture for the symmetric groups in characteristic two. Indeed, all we have to do is choose arbitrary bijections between weights (or rather, weight subgroups) and irreducibles in their blocks, which are parameterized by partitions with appropriate 2-cores.

In this paper we prove that if a table of matrices satisfying all five conditions exists, then most of its data is completely determined by a few entries.

5. 2-stability

We define the main concept of this paper.

Definition (5.1). Let λ be a partition. We call λ *2-stable* if it has the same number of rows as its 2-core.

Proposition (5.2). *Let $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)$ be a partition. The following are equivalent:*

(i) *λ is 2-stable.*

(ii) *$\lambda_t \equiv 1 (\bmod 2)$ and $\lambda_i \not\equiv \lambda_{i+1} (\bmod 2)$ for all $i = 1, \ldots, t-1$.*

(iii) *$\lambda_i \equiv t - i + 1 (\bmod 2)$ for all $i = 1, \ldots, t$.*

(iv) *λ is obtained from its 2-core by adjoining horizontal bricks to the non-empty rows of the core.*

Proof. (i) implies (ii): If λ_t were even, then the t-th row would be a string of horizontal bricks, so we could remove it and the 2-core would have at most $t-1$ rows. Thus λ_t must be odd. If λ_{t-1} were also odd, then we would be able to remove all nodes but one from λ_t, all nodes but one from its neighbour and then remove a vertical brick, which means the 2-core would have at most $t-2$

rows. Now assume that the parities of $\lambda_t, \lambda_{t-1}, \ldots, \lambda_{i+1}$ alternate. We must show that $\lambda_i \not\equiv \lambda_{i+1} (\bmod 2)$. Without loss of generality, we may assume that $\lambda_t = 1, \lambda_{t-1} = 2, \ldots, \lambda_{i+1} = t - i$ (by removing all possible horizontal bricks from the bottom row λ_t and working our way up). Note that the 2-core of λ must have t rows, so it must be the triangular partition $(t, t-1, \ldots, 1)$, and in particular, its $i+1$ row has size $t-i$. If λ_i had the same parity as λ_{i+1}, then we would be able to remove horizontal bricks from λ_i until we have $t-i$ nodes left, and then we would be able to remove a vertical brick, so that the row $i+1$ of the 2-core of λ would have at most $t-i-1$ nodes, contradicting the fact that it had exactly $t-i$ nodes.

(ii) implies (iii): We have $\lambda_t \equiv 1 (\bmod 2)$, so (iii) holds when $i = t$. Now use induction going down from $i = t$ to $i = 1$.

(iii) implies (iv): Since $\lambda_t \equiv 1 (\bmod 2)$, we can remove horizontal bricks from the last row to leave one node, then proceed to remove horizontal bricks from the previous row to leave two nodes, and work our way up until we get the triangular partition $(t, t-1, \ldots, 1)$, which is the 2-core of λ.

(iv) implies (i): If we remove the horizontal bricks that were adjoined we shall obtain the 2-core of λ, so both partitions must have the same number of rows (no bricks were added to form new rows). □

Corollary (5.3). *If λ is 2-stable, then it is also 2-regular.*

Proof. Since $\lambda_i \not\equiv \lambda_{i+1} (\bmod 2)$, no two consecutive rows can have the same size. □

Corollary (5.4). *If $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)$ is 2-stable, then the partition given by $(\lambda_1 + 1, \lambda_2 + 1, \ldots, \lambda_t + 1, 1)$ is also 2-stable.*

Proof. This follows from Proposition (5.2), part (ii). □

Remark (5.5). Note that the rows of a 2-stable partition have the same parity as the rows of its 2-core. A possible way to measure how far a partition is from being 2-stable is to count the number of its "mismatched rows", that is, the rows that have a different parity from the corresponding rows of the 2-core. The following lemma gives an estimate of how many of these rows an arbitrary partition can have.

Lemma (5.6). *Let $\mu = (\mu_1, \mu_2, \ldots, \mu_s)$ be a partition (not necessarily 2-regular) with 2-core $\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_k)$. Let $\Lambda = \{i \mid \gamma_i \not\equiv \mu_i (\bmod 2),\ 1 \leq i \leq k\}$ be the set of "mismatched" rows of μ. Then $s \geq k + 2|\Lambda|$.*

Proof. We use induction on $|\mu|$. If $|\mu| = 0$ or 1 then μ is its own 2-core and $\Lambda = \emptyset$. Now assume the result holds for all partitions of size smaller than $|\mu|$. If μ is a 2-core, then once again $\Lambda = \emptyset$ and the result holds. If μ is not a 2-core, let ν be any partition obtained from μ by removing a brick, so $|\nu| < |\mu|$ and the 2-core of ν is also γ. Let s_1 be the number of rows of ν, and $\Lambda_1 = \{i \mid \gamma_i \not\equiv \nu_i (\bmod 2),\ 1 \leq i \leq k\}$ the set of mismatched rows of ν. By the induction hypothesis, $s_1 \geq k + 2|\Lambda_1|$. There are two cases:

Case 1: We removed a horizontal brick to obtain ν from μ. Then $\Lambda_1 = \Lambda$, so $s \geq s_1 \geq k + 2|\Lambda_1| = k + 2|\Lambda|$.

Case 2: We removed a vertical brick to obtain ν from μ. Let $i, i+1$ be the rows where the vertical brick was removed. Then $\nu_i = \nu_{i+1}$. If $\nu_{i+1} > \nu_{i+2}$, then continue to remove all possible vertical bricks from rows $i, i+1$ until rows $i+1$ and $i+2$ have the same size. If $\nu_{i+2} > \nu_{i+3}$ then remove all possible vertical bricks from rows $i+1, i+2$, and continue in this manner until you reach the last two rows, $s-1, s$ (which could have been the original $i, i+1$), and simply remove them both (using vertical bricks). Let $\alpha = (\alpha_1, \ldots, \alpha_{s-2})$ be the resulting partition. Notice that α and μ have the same 2-core, $|\alpha| < |\mu|$ and α has exactly two fewer rows than μ. Let $\Lambda_2 = \{i \mid \gamma_i \neq \alpha_i (\operatorname{mod} 2),\ 1 \leq i \leq k\}$. All vertical bricks removed from any of the first k rows kept the size of two consecutive rows equal, so exactly one out of each such pair contributed to the set of mismatched rows, and the number of mismatched rows remained the same. Similarly, no vertical bricks removed from any of the rows $k+1$ through s changed the size of the set of mismatched rows (because these rows do not appear in the 2-core). The only time when the number of mismatched rows could have changed was while removing vertical bricks from the rows k and $k+1$, and this number cannot have been changed by more than one unit (depending on whether the k-th row kept its mismatched status or not). This means that $||\Lambda| - |\Lambda_2|| \leq 1$, and since α satisfies the induction hypothesis, we have

$$s - 2 \geq k + 2|\Lambda_2| \geq k + 2(|\Lambda| - 1),$$

and the result is valid for μ. □

Now we can prove our main result.

THEOREM (5.7). *Let $\lambda = (\lambda_1, \ldots, \lambda_t)$ be a 2-stable partition, and let $\mu = (\mu_1, \mu_2, \ldots, \mu_s)$ be a 2-regular partition containing λ. If $|\mu| = |\lambda| + t + 1$ and the 2-core of μ is $(t+1, t, \ldots, 1)$, then $\mu = (\lambda_1 + 1, \lambda_2 + 1, \ldots, \lambda_t + 1, 1)$, and μ is 2-stable.*

Proof. Since λ is a subpartition of μ, it is possible to write $\mu = (\lambda_1 + \alpha_1, \ldots, \lambda_t + \alpha_t, \alpha_{t+1}, \ldots, \alpha_s)$. It suffices to show that $\alpha_i \geq 1$ for all $1 \leq i \leq t+1$ since then the equality $|\mu| - |\lambda| = t+1$ forces $\alpha_i = 1$ where $1 \leq i \leq t+1 = s$. Note that $\alpha_{t+1} \geq 1$ since the 2-core of μ has $t+1$ rows. Let $\Gamma = \{i \mid \alpha_i = 0,\ 1 \leq i \leq t\}$. We must show that $\Gamma = \emptyset$. Suppose $|\Gamma| \geq 1$. Note that

$$\sum_{i=1}^{t} \alpha_i = \sum_{i \in \{1, \ldots, t\} - \Gamma} \alpha_i \geq t - |\Gamma|.$$

Since λ is 2-stable, by Proposition (5.2) (iii) it has no mismatched rows. However, the 2-core of μ is the next triangle, and all the rows of the smaller triangle must change parity, so all the rows of λ that kept their parity will be mismatched rows of μ, so Γ is a subset of the set Λ of mismatched rows of μ. By Lemma (5.6) we have $s \geq (t+1) + 2|\Lambda| \geq t + 1 + |\Gamma|$, so

$$s - t \geq |\Gamma| + 1\,.$$

Since μ is 2-regular, $1 \leq \alpha_s < \alpha_{s-1} < \cdots < \alpha_{t+1}$, so $\sum_{i=t+1}^{s} \alpha_i \geq \sum_{i=1}^{s-t} i = \frac{(s-t)(s+1-t)}{2}$, and

$$\begin{aligned} t+1 = \sum_{i=1}^{s} \alpha_i &= \sum_{i=t+1}^{s} \alpha_i + \sum_{i \in \{1,\dots,t\}-\Gamma} \alpha_i \geq \frac{(s-t)(s+1-t)}{2} + t - |\Gamma| \\ &\geq t + \frac{(|\Gamma|+1)(|\Gamma|+2)}{2} - |\Gamma| = t + \frac{|\Gamma|^2 + 3|\Gamma| + 2}{2} - |\Gamma| \\ &= t+1 + \frac{|\Gamma|^2 + |\Gamma|}{2} > t+1 \end{aligned}$$

which is a contradiction. It is now immediate that μ is 2-stable. □

If an infinite table of partitions satisfying the five conditions from Section 4 existed, then by Theorem (5.7) we see that for any 2-stable partition λ in this table, the partitions on the same row and to the right of λ are completely determined. Now we shall prove that in any row of such a table of partitions there are only finitely many partitions which are not 2-stable.

Lemma (5.8). *Let λ be a 2-regular partition with 2-core γ, and let t be the number of rows of γ. If λ is not 2-stable, then $|\lambda| - |\gamma| \geq t+1$.*

Proof. Let $\gamma = (\gamma_1 > \gamma_2 > \cdots > \gamma_t > 0 = \gamma_{t+1})$, $\lambda = (\lambda_1 > \lambda_2 > \cdots > \lambda_s)$. Since λ is not 2-stable, then $\lambda_{t+1} \geq 1 = 1 + \gamma_{t+1}$. The partition λ is 2-regular, so $\lambda_t \geq \lambda_{t+1} + 1 \geq 2 = 1 + \gamma_t$. Inductively we have that $\lambda_i \geq 1 + \lambda_{i+1} \geq 1 + (1 + \gamma_{i+1}) = 1 + \gamma_i$ for all $i = 1, \dots, t+1$. Therefore, $|\lambda| - |\gamma| \geq \sum_{i=1}^{t+1} (\lambda_i - \gamma_i) \geq t+1$. □

Corollary (5.9). *Let n be a positive integer. Then there exist only finitely many non 2-stable partitions λ such that $|\lambda| - |\gamma| \leq n$, where γ is the 2-core of λ. In particular, in any table of partitions satisfying condition 4 from Section 4, in any row of this table there are only finitely many non 2-stable partitions.*

Proof. Assume λ and γ are as above. By Lemma (5.8), the number of rows of γ is less than or equal to $n-1$, and since γ is a triangular partition, then $|\gamma| \leq (n-1)n/2$, so $|\lambda| \leq |\gamma| + n \leq \frac{(n-1)n}{2} + n = \frac{n^2+n}{2}$, and there are only finitely many partitions whose size is bounded above. The last part follows because for any row of the table, the difference between the size of any partition in that row and its 2-core is constant. □

Acknowledgements

Most of the material in this paper was part of the Ph D thesis [15] of one of the authors, directed by Peter Webb. Professor Webb suggested working on an explicit bijection between weights and simple modules for the symmetric groups in characteristic two, and he also contributed with a lot of software written in GAP [9], which was used to obtain the table of partitions from Section 4.

Received February 20, 2003

Final version received November 13, 2003

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References

[1] J. L. Alperin, Weights for Finite Groups, The Arcata Conference on Representations of Finite Groups, Proceedings of symposia in pure mathematics **47**, 369–379, Providence, R.I., Amer. Math. Soc. 1987.
[2] J. L. Alperin and P. Fong, *Weights for symmetric and general linear groups*, J. Algebra, Academic Press, New York **131** (1990), 2–22.
[3] J. B. An, 2 *weights for general linear groups*, J. Algebra **149** (1992), 500–527.
[4] J. B. An, 2 *weights for unitary groups*, Trans. Amer. Math. Soc, **339** (1993), 251–278.
[5] J. B. An, *Weights for the simple Ree groups* ${}^2G_2(q^2)$, New Zeland J. Math. **22**, (1993), 1–8.
[6] J. B. An, *Weights for the Steinberg triality groups* ${}^3D_4(q)$, Math. Z. **218** (1995), 273–290.
[7] J. B. An and M. Conder, *The Alperin and Dade conjectures for the simple Mathieu groups*, Comm. Algebra **23** (1995), 2797–2823.
[8] M. Cabanes, *Brauer Morphism between Modular Hecke Algebras*, J. Algebra, **115** (1988), 1–31.
[9] The GAP Group, GAP – Groups, Algorithms and Programming, Version 4.3, 2002 (http://www.gap-system.org)
[10] Gordon James and Andrew Mathas, *Hecke Algebras of type A with* $q = -1$, J. Algebra **184** (1996), 102–158.
[11] Gordon Douglas James, The representation theory of the symmetric groups, Springer-Verlag, Lecture Notes in Mathematics, Berlin-New York **682**, 1978.
[12] Gordon Douglas James and Adalbert Kerber, The representation theory of the symmetric group, Encyclopedia of mathematics and its applications **16**, Addison-Wesley Pub. Co., Reading, Mass. 1981.
[13] R. Knörr and G. R. Robinson, *Some remarks on a conjecture of Alperin*, J. London Math. Soc., **39**, (1989), 48–60.
[14] Okuyama, Unpublished.
[15] Luis Valero-Elizondo, On some invariants associated to simple group representations, PhD Tesis, University of Minnesota, June 1998.
[16] Luis Valero-Elizondo, *Some simple projective Brauer quotients of the simple modules for the symmetric groups in characteristic two*, J. Algebra, **236** (2001), 796–818.