

CS3001, Algorithm Design and Analysis

Tutorial 3

9. Solve the following recurrence relations. Find an exact solution for $T(n)$:

- a) $T(n) = T(n - 1) + n/3$; $T(1) = 1$
b) $T(n) = 8T(n - 1) - 15T(n - 2)$; $T(1) = 1; T(2) = 4$

10. You are given n lists S_1, \dots, S_n , each containing m real numbers. Design an divide-and-conquer algorithm to find a list R of m real numbers such that $R[i] = \max_i S_i[j]$. Give an O estimate for the runtime of the algorithm.

11. The following problem which might occur in mergesort when splitting in more than 2 parts:

The input is d sorted sequences of elements, in total they contain n elements. Design an $O(n \log d)$ algorithm to merge all sequences into one sorted sequence.

Questions 9,10 and 11 will be solved in the tutorial.

Coursework Questions

12. Let x_1, \dots, x_n be a sequence of real numbers. (Some of these might be negative!) Design an $O(n)$ algorithm to find the subsequence x_i, x_{i+1}, \dots, x_j of consecutive elements such that the product of the numbers in the subsequence is maximal possible. (The product over an empty subsequence is defined to be 1.) (4 Points)

13. Consider the following problem: The input is a set S containing n real numbers and one real number x .

- a) Design an algorithm to determine whether there are two elements of S whose sum is exactly x . The algorithm should run in time $O(n \log n)$. (3 Points)
- b) Suppose now that the set S is given in sorted order. Design an algorithm to solve the problem in time $O(n)$. (2 Points)

14. Solve the following recurrence relations. Find an exact solution for $T(n)$:

$$T(n) = n + \sum_{i=1}^{n-1} T(i); \quad T(1) = 1$$

Hint: Use index-shifting to obtain a recurrence that replaces the sum by a single term.
(4 Points)

15. The Knapsack algorithm as given in the lecture will return *one* solution, not necessarily the solution with the fewest items.

- a) Give an example of an input for which this happens. (This includes to show that it happens!) (2 Points)
- b) Modify the algorithm such that it will always return the solution with the smallest number of items. (2 Points)

Hand in solutions to questions 12-15 on Monday, November 9th, at the start of the lecture.