

CS3001, Algorithm Design and Analysis

Tutorial 2 – Coursework Questions

6) The following functions $f_i(n)$ ($1 \leq i \leq 7$) are given:

$$\begin{matrix} (e^n)^2 & 40 * n^2 - n, & 14 \log n^4, & 27 * n^2 + 3 * n + 1, \\ 14(\log n)^4, & 100n \log n, & e^{\frac{3}{2}n}, & \end{matrix}$$

- a) Arrange these functions according to O (such that $f_i(n) = O(f_{i+1})$). (Justify your assertions!) (3 Points)
- b) Assume these functions are runtimes for 7 algorithms that all solve the same problem. For which values of n (nearest integer) which algorithm has the shortest runtime? (2 Points)
- c) Assume you have an algorithm whose runtime is $T(n) = n^2 \log(n)$. For what ranges of n (order of magnitude estimate) this algorithm will run in a second, an hour and a week on a 200MHz Processor, assuming one instruction is executed in one clock cycle. (1 Points)

7) We define 4 procedures A - D as follows:

$A(n)$: **for** $i \in \{1, \dots, n\}$ **do** print "hello";**od**;
 $B(n)$: **for** $i \in \{1, \dots, n\}$ **do** $A(i)$;**od**;
 $C(n)$: **for** $i \in \{1, \dots, n\}$ **do** $B(i)$;**od**;
 $D(n)$: **for** $i \in \{1, \dots, n\}$ **do** $C(i)$;**od**;

Give estimates, as Θ classes, for the run time of each procedure. (4 Points)

8) An (undirected) graph is called a *tree* if it has only one connected component (i.e. it is connected) and it has no cycles.

- | | A tree | Not a tree |
|---|--------|------------|
| <p>Show:</p> <p>a) A connected undirected graph $G = (V, E)$ is a tree if and only if $E = V - 1$.
 Hint: You have to show: i) "tree $\Rightarrow E = V - 1$" and ii) "$E = V - 1 \Rightarrow$ tree". Use induction on V. For the induction step, show first that in both cases there must be a vertex of degree 1 (a <i>leaf</i>). (For ii), remember that $2 E = \sum_{v \in V} \deg(v)$.) Then remove this vertex. (The resulting graph will have one edge and one vertex less and will still be a tree.) (3 Points)</p> <p>b) A connected undirected graph $G = (V, E)$ is a tree if and only if for two vertices $u, v \in V$ there is exactly one path between u and v. (1 Points)</p> | | |

Hand in solutions October 9th before the tutorial.